Phil 1068 Elementary Logic: Homework 4
2nd Term 2013
DUE 25 April 4:00PM

Name:_____

Student ID #_____

Submit your problem set to Ms. Loletta Li in Room 10.13, 10/F, Run Run Shaw Tower, Centennial Campus by 4:00PM on the due date.
Make sure your problem set is timestamped (Ms. Li will do this when you turn it in).
Do not submit assignments by email.
Late penalty: 10% for each day late. Even weekends when the office is closed
Answer the questions on the problem set itself. Write neatly.
If the grader cannot read your handwriting, you will not receive credit.
Be sure that all pages of the assignment are securely stapled together.
Check the course bulletin board for announcements about the assignment.
Do your own work. This is not a collaborative assignment, and if you work with other students to solve the problems, you will fail the assignment.
If you copy your problem set, or permit others to copy, you will fail the assignment and you may fail the course.

1. (10 Marks)
True or false?
Circle ‘T’ if the statement is true.
Circle ‘F’ if the statement is false.
For this question, you should assume that φ and ψ are WFFs of MPL.

a.
T F “(Fa → ∀xFx)” is a valid PL WFF.

b.
T F “(Fx → ∃yFy)” is a valid PL WFF.

c.
T F “∃y(Fy → Gy)” entails “(∃yFy → ∃yGy).”

d.
T F “(∃yFy → ∃yGy)” entails “∃y(Fy → Gy).”

e.
T F The set of PL formulas consisting of “∀xFx” and “~∀xFx” is consistent.

f.
T F The set of PL formulas consisting of “∀xFx” and “∀x~Fx” is consistent.

g.
T F “∀xFy” is a WFF of PL.

h.

T F “P & ∀xFx” is a WFF of PL.

i.

T F The English phrase “a dog in the street” is a singular term.

j.

T F Variables are singular terms.

2. (10 Marks)

For each of the following:

Circle “valid” if it is a valid sequent.

Circle “invalid” if it is an invalid sequent.

Otherwise, don't circle anything.

a.

valid **invalid** $\sim\exists xFx \models \forall x(Fx \rightarrow Gx)$

b.

valid **invalid** $\exists x(Fx \vee Gx) \models (\exists xFx \vee \exists xGx)$

c.

valid **invalid** $(\exists xFx \vee \exists xGx) \models \exists x(Fx \vee Gx)$

d.

valid **invalid** $\forall x(Fx \leftrightarrow Gx) \models (\forall xFx \leftrightarrow \forall xGx)$

e.

valid **invalid** $(\forall xFx \leftrightarrow \forall xGx) \models \forall x(Fx \leftrightarrow Gx)$

3. (15 Marks)
Translate the following statements into PL.
Preserve as much structure as possible.
Use the following translation scheme:

Singular terms:
c: Carlos
d: Dirk

Predicates:
Px: x is petty.
Qx: x is quarrelsome.

a. “Carlos is quarrelsome only if everyone is quarrelsome.”

b. “No one petty is not quarrelsome, although someone quarrelsome is not petty.”

c. “Neither Carlos nor Dirk is petty.” [NOTE: This means the same as: “Neither Carlos is petty nor Dirk is petty.”]

d. “If everyone isn’t petty, then no one is petty.”

e. “Someone is quarrelsome if and only if he and Carlos are quarrelsome.”

4. (10 Marks)
Give a PL WFF that is logically equivalent to each of the following WFFs. Your answer must include an existential quantifier if the original WFF contains a universal quantifier, and vice versa.
(PL WFF ψ is logically equivalent to PL WFF φ if and only if φ entails ψ and ψ entails φ.)

a. ~∃x(Fx v ~Gx)

b. ∀y~(~Fx & Gx)

5. (10 Marks)
Is there an interpretation under which all the following PL WFFs are true? If yes,
then give one such interpretation. If not, explain why there is no such interpretation.

$$\exists x(Fx \,\&\, \sim Gx)$$
$$\exists x(Fx \rightarrow Gx)$$
$$\forall x \sim Gx$$
$$(Fa \vee Ga)$$

Write your interpretation, or your explanation of why there isn't one, here:

6. (10 Marks)
Are there two WFFs φ and ψ, where the set consisting of φ and ψ is inconsistent, but the conditional (φ → ψ) is true under every interpretation? If so, give examples of such a φ and ψ. If not, explain why there aren't two such WFFs.

7. (10 Marks)
Give an interpretation under which "∃x(Fx & ~Gx)" is false but "∀x(Fx → Gx)" is true.

8. (15 Marks)
All of the following sequents are derivable. Produce derivations of them in our derivation system for MPL (Note: in the readings our logic is sometimes called “PL” and sometimes “MPL” but there is no difference between the two.)

a. $\forall x \sim Fx, \forall x(Fx \vee Gx) \vdash \forall x Gx$

b. $Fa, \forall x\sim(Fx \,\&\, Gx) \vdash \sim Ga$

c. $\forall x{\sim}Gx, \exists x(Fx \rightarrow Gx) \vdash \exists x{\sim}Fx$