Phil 1068 Elementary Logic: Homework 3

2nd Term 2013

DUE 5 April 4:00PM

Name:____

Student ID #____

Submit your problem set to Ms. Loletta Li in Room 10.13, 10/F, Run Run Shaw Tower, Centennial Campus by 4:00PM on the due date.
Make sure your problem set is timestamped (Ms. Li will do this when you turn it in).
Do not submit assignments by email.
Late penalty: 10% for each day late.
Answer the questions on the problem set itself. Write neatly.
If the grader cannot read your handwriting, you will not receive credit.
Be sure that all pages of the assignment are securely stapled together.
Check the course bulletin board for announcements about the assignment.
Do your own work. This is not a collaborative assignment, and if you work with other students to solve the problems, you <u>will</u> fail the assignment.
If you copy your problem set, or permit others to copy, you <u>will</u> fail the assignment and you may fail the course.

1. (20 marks)
True or false? Circle 'T' if the statement is true. Circle 'F' if the statement is false. For this question, you should assume that φ and ψ are WFFs of SL, and that "derivable in SL" means "derivable in our SL natural deduction system."

QUESTIONS ON NEXT PAGE…

a.

T F "Everybody loves somebody or everybody does not love somebody" is a tautology.

b.

T F "Everybody loves somebody or not everybody loves somebody" is a tautology.

c.

T F If φ and ψ are inconsistent, then ~ψ ├ φ is derivable in SL.

d.

T F Every sound argument has a true conclusion.

e.

T F It is possible to add a new, sound rule to our SL natural deduction system, that allows us to prove a new WFF φ that we couldn't prove before.

f.

T F "~∃y~~Fy" is equivalent to "~~∃y~Fy."

g.

T F If φ and ψ are inconsistent, then ├ ~(φ ↔ ψ) is derivable in SL.

h.

T F If φ ├ ψ is derivable in SL, then (φ → ψ) is a tautology.

i.

T F You can prove (P → P) in any sound deductive system.

j.

T F "(~P → Q)" can be used to translate "Q unless not-P."

2. (40 marks)
Circle the correct answer.

a.

Suppose our natural deduction system is revised by removing the rules &I and &E.

Is the revised system sound? **YES** **NO**

Is the revised system complete? **YES** **NO**

b.

Suppose our natural deduction system is revised by removing the rules &I and &E and by removing the symbol "&" from SL.

Is the revised system sound? **YES** **NO**

Is the revised system complete? **YES** **NO**

c.

Suppose our natural deduction system is revised by replacing the rule →I with the rule:

If you assume ~ψ and derive ~φ, then you may write (φ → ψ) depending on everything ~φ depends on except ~ψ.

Is the revised system sound? **YES** **NO**

Is the revised system complete? **YES** **NO**

NEXT PAGE…

d.

Suppose our natural deduction system is revised <u>by replacing</u> the rule →I with the rule:

If you derive φ and you derive ψ, then you may write (φ → ψ) depending on everything ψ depends on except φ.

Is the revised system sound? **YES** **NO**

Is the revised system complete? **YES** **NO**

e.

Suppose our natural deduction system is revised <u>by replacing</u> the rule →I with the rule:

If you assume φ and you derive ψ, then you may write (φ → ψ) depending on everything ψ depends on.

Is the revised system sound? **YES** **NO**

Is the revised system complete? **YES** **NO**

f.

Suppose our natural deduction system is revised <u>by replacing</u> the rule →I with the rule:

If you assume ψ and you derive φ, then you may write (φ → ψ) depending on everything φ depends on, except ψ.

AND <u>by replacing</u> the rule →E with the rule:

If you derive (φ → ψ) and you derive ψ, then you may write down φ, depending on everything (φ → ψ) and ψ depend on.

Is the revised system sound? **YES** **NO**

Is the revised system complete? **YES** **NO** g.

Suppose our natural deduction system is revised by adding the rule

If you derive $(\varphi \leftrightarrow \psi)$

and you derive $(\varphi \rightarrow \chi)$

and you derive $(\psi \rightarrow \chi)$,

then you may write χ depending on everything that

$(\varphi \leftrightarrow \psi)$, $(\varphi \rightarrow \chi)$, and $(\psi \rightarrow \chi)$ depend on.

Is the revised system sound? **YES** **NO**

Is the revised system complete? **YES** **NO**

h.

Suppose our natural deduction system is revised by adding the rule

If you derive $\sim(\varphi \leftrightarrow \psi)$

and you derive $(\varphi \rightarrow \chi)$

and you derive $(\psi \rightarrow \chi)$,

then you may write χ depending on everything that

$\sim(\varphi \leftrightarrow \psi)$, $(\varphi \rightarrow \chi)$, and $(\psi \rightarrow \chi)$ depend on.

Is the revised system sound? **YES** **NO**

Is the revised system complete? **YES** **NO**

NEXT PAGE…

3. (20 marks)

Translate the following statements into PL.

Preserve as much structure as possible.

Use “Bx” to translate “x is black,” “Cx” to translate “x is a cat,” and “Dx to translate “x is dangerous.”

a. “All cats are dangerous.”

b. “Nothing is dangerous.”

c. “Some black cats are not dangerous.”

d. “Not everything is not a cat.”

e. “If everything is dangerous, then some cats are dangerous.”

NEXT PAGE…

4. (20 marks)

Translate the following PL WFFs into English.

Use the same translation scheme as in problem #3.

a. $\forall z(Cz \,\&\, Dz)$

b. $\exists x(Bx \,\&\, Cx)$

c. $\exists y(By \rightarrow Cy)$

d. $\sim\exists xBx$

e. $(\exists xBx \rightarrow \forall zCz)$