

Problem Set 2
Phil 1068 Elementary Logic
2nd Term 2013
Due 28 February 2013 by 4:00PM

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Submit your problem set to Ms. Loletta Li in Room 10.13, 10/F, Run Run Shaw Tower, Centennial Campus by 4:00PM on the due date.

Make sure your problem set is timestamped (Ms. Li will do this when you turn it in).

Do not submit assignments by email.

Late penalty: 10% for each day late.

Answer the questions on the problem set itself. Write neatly.

If the grader cannot read your handwriting, you will not receive credit.

Be sure that all pages of the assignment are securely stapled together.

Check the course bulletin board for announcements about the assignment.

Do your own work.

If you copy your problem set, or permit others to copy, you will fail the assignment and you may fail the course.

1. (10 marks)

True or false? Circle 'T' if the statement is true. Circle 'F' if the statement is false.

For this question, you should assume that ϕ , ψ , and χ are WFFs of SL.

a.

T ϕ entails " $\sim(P \leftrightarrow \sim P)$."

b.

T If ϕ is consistent with ψ , and ψ entails χ , then ϕ is consistent with χ .

c.

T If ϕ , ψ , and χ form an inconsistent set of WFFs, then ϕ and ψ together entail $\sim\chi$.

d.

T “ $\sim(P \rightarrow \sim Q)$ ” is logically equivalent to “ $(P \& Q)$.”

e.

F If ϕ entails ψ , and ϕ entails χ , then ψ entails χ .

2. (20 marks)

Translate the following statements into SL. Preserve as much structure as possible. Use the following translation scheme:

A: Alvin is angry.

B: Bill is benevolent.

C: Clara is cute.

D: Diana is disgusting.

a. If (if Clara is cute, then Diana is disgusting), then Alvin is angry.

Note: I've put some brackets here to disambiguate the sentence for you. This is not an acceptable sentence of English, but it does have a sensible translation into SL!

$((C \rightarrow D) \rightarrow A)$

b. Alvin's being angry is necessary for Clara's not being cute.

$(\sim C \rightarrow A)$

c. Although Clara is cute, Diana is disgusting.

$(C \& D)$

d. Whenever Diana is disgusting, (Alvin is angry, unless Clara is cute).

Note: *I've put some brackets here to disambiguate the sentence for you.*

(D \rightarrow (A \vee C))

e. Bill is benevolent only if Diana is not disgusting.

(B \rightarrow \sim D)

3. (10 marks)

Suppose at least one of the following two statements is false:

a. Sam has \$10.

b. Sam buys a coke whenever he has \$10.

Translate each of the two statements into SL, preserving as much structure as possible. Be sure to write down your translation scheme.

H: Sam has \$10.

B: Sam buys a coke.

a. H

b. (H \rightarrow B)

Did Sam buy a coke?

It is impossible to tell, given the information in the question.

4. (20 marks)

For each of the following:

Circle “tautology” if it is a WFF of SL that is a tautology.

Circle “contingent” if it is a contingent WFF of SL.

Circle “inconsistent” if it is an inconsistent WFF of SL.

Otherwise, don't circle anything.

QUESTIONS ON NEXT PAGE...

a.

$$\sim(P \rightarrow \sim P)$$

inconsistent contingent tautology

b.

$$(\sim(P \vee \sim Q) \leftrightarrow (\sim P \& Q))$$

inconsistent contingent tautology

c.

$$(((P \rightarrow Q) \rightarrow P) \rightarrow Q)$$

inconsistent contingent tautology

d.

$$((P \rightarrow Q) \vee (Q \rightarrow P))$$

inconsistent contingent tautology

e.

$$((Q \& P) \vee (\sim Q \& P))$$

inconsistent contingent tautology

f.

$$((P \rightarrow Q) \rightarrow ((Q \rightarrow R) \rightarrow (P \rightarrow R)))$$

inconsistent contingent tautology

g.

$$(P \ \& \ (Q \ \rightarrow \ R)) \leftrightarrow ((P \ \& \ \sim Q) \vee (P \ \& \ R))$$

inconsistent contingent tautology

h.

$$((\sim P \ \& \ (\sim Q \ \& \ R)) \leftrightarrow (P \ \vee \ (Q \ \vee \ \sim R)))$$

inconsistent contingent tautology

i.

$$((P \ \rightarrow \ Q) \ \rightarrow \ (\sim P \ \rightarrow \ \sim Q))$$

inconsistent contingent tautology

j.

$$(((P \ \leftrightarrow \ Q) \ \leftrightarrow \ R) \leftrightarrow ((P \ \leftrightarrow \ R) \ \leftrightarrow \ Q))$$

inconsistent contingent tautology

5. (40 marks) All of the following sequents are derivable using the SL natural deduction system for this course. Derive them.

$$a. (P \ \rightarrow \ (Q \ \rightarrow \ R)) \ \vdash \ ((P \ \& \ Q) \ \rightarrow \ R)$$

1	1. $(P \ \rightarrow \ (Q \ \rightarrow \ R))$	A
2	2. $(P \ \& \ Q)$	A
2	3. P	2 &E
2	4. Q	3 &E
1,2	5. $(Q \ \rightarrow \ R)$	1,3 \rightarrow E
1,2	6. R	5,4 \rightarrow E
1	7. $((P \ \& \ Q) \ \rightarrow \ R)$	2,6 \rightarrow I

b. $((P \vee Q) \& ((Q \vee R) \& S)) \vdash (S \& ((Q \vee R) \& (P \vee Q)))$

1	1. $((P \vee Q) \& ((Q \vee R) \& S))$	A
1	2. $(P \vee Q)$	1 &E
1	3. $((Q \vee R) \& S)$	1 &E
1	4. $(Q \vee R)$	3 &E
1	5. S	3 &E
1	6. $((Q \vee R) \& (P \vee Q))$	4,2 &I
1	7. $(S \& ((Q \vee R) \& (P \vee Q)))$	5,6 &I

c. $(P \leftrightarrow \sim P), \sim P \vdash (P \& \sim P)$

1	1. $(P \leftrightarrow \sim P)$	A
2	2. $\sim P$	A
1	3. $((P \rightarrow \sim P) \& (\sim P \rightarrow P))$	1 \leftrightarrow E
1	4. $(\sim P \rightarrow P)$	3 &E
1,2	5. P	2,4 \rightarrow E
1,2	6. $(P \& \sim P)$	5,2 &I

d. $\vdash (P \leftrightarrow P)$

1	1. P	A
	2. $(P \rightarrow P)$	1,1 \rightarrow I
	3. $((P \rightarrow P) \& (P \rightarrow P))$	2,2 &I
	4. $(P \leftrightarrow P)$	3 \leftrightarrow I

e. $P \vdash (Q \rightarrow P)$

1	1. P	A
2	2. Q	A
1	3. $(Q \rightarrow P)$	2,1 \rightarrow I