Logic for Philosophers
Problem Set 2
Due April 18th

Name: ____

Student ID#____

All of the following sequents are derivable. Provide derivations for them.

1. $\forall xFx, \forall x(Fx \rightarrow Gx) \vdash \forall xGx$

1	1. $\forall xFx$	A
2	2. $\forall x(Fx \rightarrow Gx)$	A
1	3. Fa	1 $\forall$E
2	4. $(Fa \rightarrow Ga)$	2 $\forall$E
1,2	5. Ga	3, 4 $\rightarrow$E
1,2	6. $\forall xGx$	5 $\forall$I

2. $Fa, \forall x(Gx \rightarrow {\sim}Fx) \vdash {\sim}Ga$

1	1. Fa	A
2	2. $\forall x(Gx \rightarrow {\sim}Fx)$	A
3	3. Ga	A
2	4. $(Ga \rightarrow {\sim}Fa)$	2 $\forall$E
2,3	5. ${\sim}Fa$	3, 4 $\rightarrow$E
1,2,3	6. $(Fa \,\&\, {\sim}Fa)$	1, 5 &E
1,2	7. ${\sim}Ga$	3, 6 ~I

3. $\forall x(Fx \rightarrow Gx), \exists xFx \vdash \exists xGx$

1	1. $\forall x(Fx \rightarrow Gx)$	A
2	2. $\exists xFx$	A
3	3. Fa	A
1	4. $(Fa \rightarrow Ga)$	1 $\forall$E

1,3	5. Ga	3, 4 →E
1,3	6. ∃xGx	5 ∃I
1,2	7. ∃xGx	2, 3, 6 ∃E

4. ∀x(Fx → Gx) ├ (Fa → Ga)

1	1. ∀x(Fx → Gx)	A
1	1. (Fa → Ga)	1 ∀E

5. ∀x(Fx v Gx), ∀x~Fx ├ ∀xGx

1	1. ∀x(Fx v Gx)	A
2	2. ∀x~Fx	A
1	3. (Fa v Ga)	1 ∀E
2	4. ~Fa	2 ∀E
1,2	5. Ga	3, 4 vE
1,2	6. ∀xGx	5 ∀I

6. ∀x(Fx → Gx), ∃x(Fx & Hx) ├ ∃x(Fx & Gx)

1	1. ∀x(Fx → Gx)	A
2	2. ∃x(Fx & Hx)	A
3	3. (Fa & Ha)	A
3	4. Fa	3 &E
1	5. (Fa → Ga)	1 &E
1,3	6. Ga	4, 5 →E
1,3	7. (Fa & Ga)	4, 6 &I
1,3	8. ∃x(Fx & Gx)	7 ∃I
1,2	9. ∃x(Fx & Gx)	2, 3, 8 ∃E

7. ∀x(Fx → Gx) ├ (~∃xGx → ~∃xFx)

1	1. ∀x(Fx → Gx)	A
2	2. ~∃xGx	A
3	3. ∃xFx	A
4	4. Fa	A

1	5. (Fa → Ga)	1 ∀E
1,4	6. Ga	4, 5 →E
1,4	7. ∃xGx	6 ∃I
1,2,4	8. (∃xGx & ~∃xGx)	2, 7 &I
1,2, 3	9. (∃xGx & ~∃xGx)	3, 4, 8 ∃E
1,2	10. ~∃xFx	3, 9 ~I
1	11. (~∃xGx → ~∃xFx)	2, 10 →I

8. (∀xFx & ∀xGx) ├ ∀x(Fx & Gx)

1	1. (∀xFx & ∀xGx)	A
1	2. ∀xFx	1 &E
1	3. ∀xGx	1 &E
1	4. Fa	2 ∀E
1	5. Ga	3 ∀E
1	6. (Fa & Ga)	4, 5 &I
1	7. ∀x(Fx & Gx)	6 ∀I

9. (∃xFx v ∃xGx) ├ ∃x(Fx v Gx)

1	1. (∃xFx v ∃xGx)	A
2	2. ~∃x(Fx v Gx)	A
3	3. ~∃xFx	A
1,3	4. ∃xGx	1, 3 vE
5	5. Ga	A
5	6. (Fa v Ga)	5 vI
5	7. ∃x(Fx v Gx)	6 ∃I
1,3	8. ∃x(Fx v Gx)	4, 5, 7 ∃E
1,2,3	9. (∃x(Fx v Gx) & ~∃x(Fx v Gx))	2, 8 &I
1,2	10. ∃xFx	3, 9 ~E
11	11. Fa	A
11	12. (Fa v Ga)	11 vI
11	13. ∃x(Fx v Gx)	12 ∃I
1,2	14. ∃x(Fx v Gx)	10, 11, 13 ∃E
1,2	15. (∃x(Fx v Gx) & ~∃x(Fx v Gx))	2, 14 &I
1	16. ∃x(Fx v Gx)	2, 15 ~E

10. ∀xFx ├ ~∃x~Fx

1	1. ∀xFx	A
2	2. ∃x~Fx	A
3	3. ~Fa	A
1	4. Fa	1 ∀E
1,3	5. (Fa & ~Fa)	3, 4 &I
3	6. ~∀xFx	1 ~I
1,3	7. (∀xFx & ~∀xFx)	1, 6 &I
1,2	8. (∀xFx & ~∀xFx)	2, 3, 7 ∃E
1	9. ~∃x~Fx	2, 8 ~I