Elementary Logic: Homework 3

DUE 29 March 4:00PM

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1. (15 marks)

True or false? Circle `T' if the statement is true. Circle `F' if the statement is false. For this question, you should assume that ' and are WFFs of SL, and that \the system" is our SL natural deduction system.

a.
T F "((P v P) ↔ P)" is derivable with no dependencies in our natural deduction system.

b.
T F If an argument has true premises and a true conclusion, then it is sound.

c.
T F If the rule ~I is removed from the system, we cannot prove P $\vdash$ ~~P.

d.
T F "believe" is a truth-functional connective in English.

e.
T F If φ is a tautology, then $\psi \vdash \varphi$ is derivable in our natural deduction system.

f.
T F If a natural deduction system allows us to derive $\vdash \varphi$, for some contingent WFF φ, then the system is unsound.

g.
T F "Someone is happy and it is not the case that someone is happy" is a contradiction.

h.
T F If a valid argument has a false conclusion, then all of its premises are false.

i.
T F Suppose we introduce a new truth-functional connective "\$" and you are told that P is equivalent to "\$\$P". Then it follows that "\$P" is equivalent to "~P".

j.
T F Natural deduction systems that are not sound are also not complete.

k.
T F If $\varphi \vdash (\psi$ & $\sim\psi)$ is derivable in our natural deduction system, then φ is inconsistent.

2. (28 marks)

Circle your answer.

a.

Suppose our natural deduction system is revised so that it contains only the following rule:

At any point in a derivation, you may write down φ, depending on nothing.

Is the revised system sound? **YES** **NO**

Is the revised system complete? ~~**YES**~~ **NO**

b.

Suppose our natural deduction system is revised by adding the following rule:

At any point in a derivation, you may write down φ, depending on nothing.

Is the revised system sound? **YES** **NO**

Is the revised system complete? YES NO

c.

Suppose our natural deduction system is revised by adding the following rule:

At any point in a derivation, you may write down (φ v ~φ), depending on nothing.

Is the revised system sound? YES NO

Is the revised system complete? YES NO

d.

Suppose our natural deduction system is revised by adding the following rule:

If you have derived ~(φ & ψ),

and you have derived φ,

then you can write down ~ψ depending on everything that

~(φ & ψ) depends on.

Is the revised system sound? YES NO

Is the revised system complete? YES NO

e.

Suppose our natural deduction system is revised by adding the following rule:

If you have derived (φ v ψ),

and you have derived (φ → χ),

and you have derived (ψ → χ),

then you can write down χ depending on everything that

(φ v ψ), (φ → χ), and (ψ → χ) depend on.

Is the revised system sound? YES NO

Is the revised system complete? YES NO

f.

Suppose our natural deduction system is revised by removing the rule PC.

Is the revised system sound? YES NO

Is the revised system complete? YES NO

g.

Suppose our natural deduction system is revised by removing the rule &E.

Is the revised system sound? YES NO

Is the revised system complete? YES NO

3. (15 marks)

Translate the following statements into MPL.

Preserve as much structure as possible.

Use "S" to translate "is on sale" and "E" to translate "is expensive."

a. Something is on sale.

$\exists x Sx.$

b. Nothing on sale is expensive.

$\sim\exists x (Sx \& Ex)$

c. If something is on sale, then not everything is expensive.

$(\exists x Sx \rightarrow \sim\forall y Ey)$

d. Everything is either on sale or expensive.

$\forall x (Sx \vee Ex)$

e. If something is on sale, then it is not expensive.

$\forall x (Sx \rightarrow \sim Ex)$