Problem Set 4
Phil 1068 Elementary Logic
Due April 18th

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1. (10 marks) True or false?
Circle 'T' if the statement is true.
Circle 'F' if the statement is false.
For this question, you should assume that φ and ψ are WFFs of MPL.

(a)
T F In the statement "Henry is happy," "is happy" is the subject and "Henry" is the predicate.
(b)
T F "∃x∃yRyx" is a WFF of MPL.
(c)
T F You cannot use the truth-table method to determine whether WFFs of MPL are consistent.
(d)
T F "(∃xFx → ∃yFy)" is a valid MPL WFF.
(e)
T F "(∃xFx → ∃xGx)" is a valid MPL WFF.
(f)
T F For any φ, φv/c is a WFF of MPL.
(g)
T F For any φ, φc/v is a WFF of MPL.
(h)
T F "∃x(Fx v Gx)" entails "∃xFx".
(i)
T F "(∃xFx & ∃xGx)" entails "∃x(Fx & Gx)".
(j)
T F The set of MPL formulas consisting of ∃xFx and ~∃xFx is consistent.

2. (10 marks)
For each of the following:
Circle "valid" if it is a valid sequent.
Circle "invalid" if it is an invalid sequent.
Otherwise, don't circle anything.

(a)
valid invalid (∀xEx & Sb) ⊨ (Eb & Sb)
(b)
valid invalid ∀x(Px & Qx) ⊨ (∀xPx & ∀xQx)
(c)
valid invalid (∃xPx & ∃xQx) ⊨ ∃x(Px & Qx)
(d)
valid invalid ∃x(Px & Qx) ⊨ (∃xPx & ∃Qx)
(e)
valid invalid (∀xPx & ∀xQx) ⊨ ∀x(Px & Qx)

3. (15 marks)
Translate the following statements into MPL.
Preserve as much structure as possible.
Use the following translation scheme:

a: Alice
b: Betty
Fx: x is friendly
Gx: x is grateful

(a) "If someone is friendly, Alice is friendly."

(∃xFx → Fa)

(b) "Someone friendly is not grateful, but everyone grateful is friendly."

(∃x(Fx & ~Gx) & ∀x(Gx → Fx))

(c) "Alice is friendly unless Betty is not friendly."

(Fb → Fa)

(d) "Someone is such that if they are friendly then Betty is grateful."

∃x(Fx → Gb)

(e) "Everyone is grateful unless nobody is grateful."

$(\exists x Gx \rightarrow \forall x Gx)$

4. (10 marks)
Give an MPL WFF that is logically equivalent to each of the following WFFs. Your answer must include an existential quantifier if the original WFF contains a universal quantifier, and vice versa.
(MPL WFF ψ is logically equivalent to MPL WFF φ if and only if φ entails ψ and ψ entails φ.)

(a) $\exists x \sim(Fx \leftrightarrow Gx)$

$\sim\forall x(Fx \leftrightarrow Gx)$.

(b) $\sim\forall y(\sim Fx \rightarrow Gx)$

~~$\exists y$~~ not a WFF.

5. (10 marks)
Is there an interpretation under which all the following MPL WFFs are true? If yes, then give one such interpretation. If not, explain why there is no such interpretation.

$\forall x((Ax \& Bx) \rightarrow \sim Cx)$
$\exists x(Bx \& Cx)$
$\exists x(\sim Cx \leftrightarrow Ax)$
Ca

Domain: the set of all animals.
Ax : x is a mammal.
Bx : x is a vertebrate.
Cx : x is a bird.
a : a penguin in the Ocean Park.

6. (10 marks)
Is there a consistent WFF that is true under every interpretation? If so, give such a WFF. If not, explain why there is no such WFF.

$(\forall x Fx \vee \sim\forall x Fx)$

7. (10 marks)
Give an interpretation under which $\exists x(Fx \vee \sim Gx)$ is false and $\forall x(Gx \rightarrow Fx)$ is true

Impossible.

8. (15 marks)
All of the following sequents are derivable. Produce derivations of them.

(a) $\forall xFx, \forall x(Fx \rightarrow Gx) \vdash \forall xGx$

1	1.	$\forall x Fx$.	A.
2	2.	$\forall x(Fx \rightarrow Gx)$	A.
1	3	Fa	1. ∀E
2	4.	$(Fa \rightarrow Ga)$	2 ∀E
1.2	5	Ga	3,4 →E
1,2	6	$\forall x Gx$.	5 ∀I.

(b) Fa, ∀x(Gx → ~Fx) ⊢ ~Ga

1.	1.	Fa	A.
2	2.	$\forall x(Gx \rightarrow \sim Fx)$	A.
3	3.	Ga	A.
2	4	$(Ga \rightarrow \sim Fa)$	2 ∀E
2.3.	5	$\sim Fa$	3,4 →E
1,2,3.	6	$(Fa \,\&\, \sim Fa)$	1,5 &I.
1.2.	7	$\sim Ga$	3.6 ~I.

(c) ∀x(Fx → Gx), ∃xFx ⊢ ∃xGx

1	1	$\forall x(Fx \rightarrow Gx)$	A.
2	2.	$\exists x Fx.$	A.
1	3	$(Fa \rightarrow Ga)$	1. ∀E.
4.	4	Fa	A.
1.4.	5	Ga	3,4 →E
1,4.	6	$\exists x Gx.$	5 ∃I.
1,2	7.	$\exists x Gx.$	2,4,6 ∃E