Problem Set 4
Phil 1068 Elementary Logic
Due April 18th

Name:_____

Student ID#:_____

1. (10 marks) True or false?

Circle 'T' if the statement is true.

Circle 'F' if the statement is false.

For this question, you should assume that φ and ψ are WFFs of MPL.

(a)

T F In the statement "Henry is happy," "is happy" is the subject and "Henry" is the predicate.

(b)

T F "∃x∃yRyx" is a WFF of MPL.

(c)

T F You cannot use the truth-table method to determine whether WFFs of MPL are consistent.

(d)

T F "(∃xFx → ∃yFy)" is a valid MPL WFF.

(e)

T F "(∃xFx → ∃xGx)" is a valid MPL WFF.

(f)

T F For any φ, φv/c is a WFF of MPL.

(g)

T F For any φ, φc/v is a WFF of MPL.

(h)

T F "∃x(Fx v Gx)" entails "∃xFx".

(i)

T F "(∃xFx & ∃xGx)" entails "∃x(Fx & Gx)".

(j)

T F The set of MPL formulas consisting of ∃xFx and ~∃xFx is consistent.

2. (10 marks)
For each of the following:
Circle "valid" if it is a valid sequent.
Circle "invalid" if it is an invalid sequent.
Otherwise, don't circle anything.

(a)
valid   invalid   (∀xEx & Sb) ⊨ (Eb & Sb)
(b)
valid   invalid   ∀x(Px & Qx) ⊨ (∀xPx & ∀xQx)
(c)
valid   invalid   (∃xPx & ∃xQx) ⊨ ∃x(Px & Qx)
(d)
valid   invalid   ∃x(Px & Qx) ⊨ (∃xPx & ∃Qx)
(e)
valid   invalid   (∀xPx & ∀xQx) ⊨ ∀x(Px & Qx)

3. (15 marks)
Translate the following statements into MPL.
Preserve as much structure as possible.
Use the following translation scheme:

a: Alice
b: Betty
Fx: x is friendly
Gx: x is grateful

(a) "If someone is friendly, Alice is friendly."

(b) "Someone friendly is not grateful, but everyone grateful is friendly."

(c) "Alice is friendly unless Betty is not friendly."

(d) "Someone is such that if they are friendly then Betty is grateful."

(e) "Everyone is grateful unless nobody is grateful."

4. (10 marks)
Give an MPL WFF that is logically equivalent to each of the following WFFs. Your answer must include an existential quantifier if the original WFF contains a universal quantifier, and vice versa.
(MPL WFF ψ is logically equivalent to MPL WFF φ if and only if φ entails ψ and ψ entails φ.)

(a) ∃x~(Fx ↔ Gx)

(b) ~∀y(~Fx → Gx)

5. (10 marks)
Is there an interpretation under which all the following MPL WFFs are true? If yes, then give one such interpretation. If not, explain why there is no such interpretation.

∀x((Ax & Bx) → ~Cx)
∃x(Bx & Cx)
∃x(~Cx ↔ Ax)
Ca

6. (10 marks)

Is there a consistent WFF that is true under every interpretation? If so, give such a WFF. If not, explain why there is no such WFF.

7. (10 marks)

Give an interpretation under which ∃x(Fx v ~Gx) is false and ∀x(Gx → Fx) is true

8. (15 marks)

All of the following sequents are derivable. Produce derivations of them.

(a) ∀xFx, ∀x(Fx → Gx) ⊢ ∀xGx

(b) Fa, ∀x(Gx → ~Fx) ⊢ ~Ga

(c) ∀x(Fx → Gx), ∃xFx ⊢ ∃xGx