**Midterm: Elementary Logic (1068): 8 March 2012**

Student ID Number: ____________________

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1. (15 Marks)
True or False. Circle 'T' if the statement is true and 'F' if the statement is false.
For this question, assume that φ is a WFF of SL

a.
**T** **F** The premises of a sound argument are true.

b.
**T** **F** Some invalid arguments are sound.

c.
**T** **F** "I like cheese!" is a statement.

d.
**T** **F** If A entails B, then "(A → B)" is a tautology.

e.
**T** **F** If A doesn't entail B, then "(A & ~B)" is inconsistent.

f.
**T** **F** The scope of "~" in "~P" is "~P".

g.
**T** **F** Every disjunction is contingent.

h.
**T** **F** If A is logically equivalent to B and B is logically equivalent to C, then A is logically equivalent to C.

i.
**T** **F** "(P ↔ ~P)" is inconsistent.

j.
**T** **F** φ has either 0 parentheses or an even number of parentheses.



2. (5 marks)
Write down a valid SL WFF.

⊢ (A → A)

3. (15 marks)
Make a truth-table for each of the following WFFs

a. ~(P → ~Q)

| P | Q | ~(P → ~Q) |
|---|---|---|
| T | T | T |
| T | F | F |
| F | T | F |
| F | F | F |

b. (P ↔ (Q v R))

| P | Q | R | (P ↔ (Q v R)) |
|---|---|---|---|
| T | T | T | T |
| T | T | F | T |
| T | F | T | T |
| T | F | F | F |
| F | T | T | F |
| F | T | F | F |
| F | F | T | F |
| F | F | F | T |

c. (P v ~~P)

| P | (P v ~~P) |
|---|---|
| T | T |
| F | F |

d. ~((P & Q) v ~R)

| P | Q | R | ~((P&Q) v ~R) |
|---|---|---|---|
| T | T | T | F |
| T | T | F | F |
| T | F | T | T |
| T | F | F | F |
| F | T | T | T |
| F | T | F | F |
| F | F | T | T |
| F | F | F | F |

e. (((P → Q) → P) → P)

| P | Q | (((P→Q)→P)→P) |
|---|---|---|
| T | T | T |
| T | F | T |
| F | T | T |
| F | F | T |

4. (10 marks) Fill in the blanks with a WFF to make correct truth-tables. **Each WFF you write must be a biconditional.** -1 for a pair of missing brackets

OR any logically equivalent biconditional.

a.

| P | Q | (P ↔ (Q & ~Q)) |
|---|---|---|
| T | T | F |
| T | F | F |
| F | T | T |
| F | F | T |

b.

| P | Q | ((P v ~P) ↔ (~P & Q)) |
|---|---|---|
| T | T | F |
| T | F | F |
| F | T | T |
| F | F | F |



full marks will only be given with appropriate explanation.

5. (10 marks) Suppose that a new two-place connective '#' is added to SL. You are informed that "(A & ~B)" entails "(A # B)" and that "(A # A)" is inconsistent.
If possible, complete the following truth-table.
If it is not possible write "not possible", and explain why.

| A | B | (A # B) |
|---|---|---|
| T | T | F |
| T | F | T |
| F | T | ? |
| F | F | F |

Room for explanation (if necessary):

It's impossible for us to complete the truth table of (A#B) since "(A#A) is inconsistent" only tells us that the first two rows of the table are both "F" while "(A&~B)" entails "(A#B)" only tells us that the second row is T.

6. (5 marks) Which of the following is a valid argument. Circle 'Yes' for all the valid arguments and 'No' for all the invalid ones. Circle nothing if the argument is neither valid nor invalid.

a.
(Premise) John is happy if and only if Bill is not happy.
(Conclusion) John is not happy if and only if Bill is happy.
**Yes** **No**

b.
(Premise) Fred eats worms.
(Premise) If Fred doesn't eat worms, then Fred eats worms.
(Conclusion) Pandas have seventeen legs.
**Yes** **No**

c.
(Premise) Fred eats worms.
(Premise) If Fred eats worms, then Fred doesn't eat worms.
(Conclusion) Pandas have seventeen legs.
**Yes** **No**

7. (10 marks) Translate the following statements into SL, preserving as much structure as possible. Be sure to write down you translation scheme.

a. "All you need is love."

A : All you need is love.

A

b. "If I had a million dollars, I wouldn't buy a golden house and a rocket car."

M : I had a million dollars.
G : I would buy a golden house.
R : I would buy a rocket car.

(M → ~(G & R))

c. "John's having a party tomorrow is a sufficient condition for George to begin drinking again."

J : John's having a party tomorrow.

G : George to begin drinking again.

(J → G)



d. "You can have set lunch A or set lunch B for $98 (but not both of them)."

A : You can have set lunch A for $98.
B : You can have set lunch B for $98.

((A v B) & ~(A & B))

e. "If God exists then since God is all good he wants us to be happy; and if God wants us to be happy, then since God is all powerful, he will make us happy."

E : God exists.
G : God is all good
W : God wants us to be happy
P : God is all powerful
M : God will make us happy

~~G & E~~

G ⊦(E → W)

P ⊦ (W → M)

8. (30 marks) All of the following sequents are derivable. Prove them using the natural deduction system for this course.

−1 for a pair of missing brackets
−2 for minor mistakes regarding dependence
0 for missing rule.

1. $(((\sim P \& Q) \& R) \rightarrow (Q \vee P)) \vdash (\sim P \rightarrow ((Q \& R) \rightarrow (Q \vee P)))$

| | | | |
|---|---|---|---|
| 1 | 1 | (((~P & Q) & R) → (Q v P)) | A |
| 2 | 2. | ~P | A |
| 3. | 3. | (Q & R) | A |
| 3 | 4 | Q | 3 &E |
| 3 | 5 | R | 3 &E |
| 2.3 | 6. | (~P & Q) | 2, 4 &I. |
| 2.3 | 7. | ((~P & Q) & R) | 5. 6 &I. |
| 1, 2, 3 | 8. | (Q v P) | 1, 7. →E |
| 1, 2 | 9. | ((Q & R) → (Q v P)) | 3, 8 →I. |
| 1. | 10 | (~P → ((Q & R) → (Q v P))) | 2, 9 →I. |

2. Q ⊢ ((P & Q) ↔ P)

| | | | | |
|---|---|---|---|---|
| 1 | 1 | $Q$ | ~~A~~. | A |
| 2 | 2 | $P$ | ~~A~~. | A. |
| 1.2 | 3. | $(P \& Q)$ | | 1,2 &I. |
| 1. | 4. | $(P \rightarrow (P \& Q))$ | | 2,3 →I. |
| 5 | 5. | $(P \& Q)$ | | A. |
| 5 | 6. | $P$ | | 5. &E. |
| | 7. | $((P \& Q) \rightarrow P)$ | | 5,6 →I. |
| 1. | 8. | $((P \rightarrow (P \& Q)) \& ((P \& Q) \rightarrow P))$ | | 4.7 &I. |
| 1. | 9. | $((P \& Q) \leftrightarrow P)$ | | 8 ↔I. |

3. ⊢ (P v ~P)

| | | | |
|---|---|---|---|
| 1 | 1 | $\sim(P \vee \sim P)$ | A. |
| 2. | 2 | $P$ | A. |
| 2. | 3 | $(P \vee \sim P)$ | 2. vI. |
| 1,2. | 4 | $(\sim(P \vee \sim P) \& (P \vee \sim P))$ | 1,3 &I. |
| 1. | 5 | $\sim P$ | 2,4 ~I |
| 1. | 6 | $(P \vee \sim P)$ | 5 vI. |
| 1 | 7. | $(\sim(P \vee \sim P) \& (P \vee \sim P))$ | 1.6 &I. |
| | 8 | $(P \vee \sim P)$ | 1,7 ~E |