

**Problem Set 2**  
**Phil 1068: Elementary Logic**  
**Due 22 February**

Name: \_\_\_\_\_

Student ID Number: \_\_\_\_\_

Submit your problem set to Ms. Loletta Li in Main Building 312.

Make sure your problem set is timestamped.

Do not submit assignments by email.

Late penalty: 10% for each day late.

~~This problem set will not be accepted after 21 February.~~ **IGNORE THIS**

Answer the questions on the problem set itself. Write neatly.

If the grader cannot read your handwriting, you will not receive credit.

Be sure that all pages of the assignment are securely stapled together.

Check the course bulletin board for announcements about the assignment.

Do your own work.

If you copy your problem set, or permit others to copy, you may fail the course.

1. (10 marks)

True or false? Circle 'T' if the statement is true. Circle 'F' if the statement is false.

For this question, you should assume that  $\phi$ ,  $\psi$ , and  $\chi$  are WFFs of SL.

T F '( $P \vee Q$ )' is logically equivalent to '( $\sim P \rightarrow Q$ )'

T F If  $\phi$  entails  $\psi$ , and  $\psi$  entails  $\chi$ , then  $\phi$  entails  $\chi$

T F '( $P \& \sim P$ )' entails  $\phi$

T F ' $\sim(P \& \sim Q)$ ' is inconsistent

T F '( $P \rightarrow Q$ )' is contingent

2. (20 marks)

Translate the following statements into SL. Preserve as much structure as possible.  
Use the following translation scheme:

P: Paul is grumpy

Q: Quentin is being obnoxious

R: Ralph is happy

S: Susan is happy

(a) Ralph is happy when Susan is, unless Quentin is being obnoxious.

(b) Quentin being obnoxious is sufficient but not necessary for Paul to be grumpy.

(c) Either Ralph or Susan is happy, but not both of them.

(d) Unless Ralph is happy, Paul is grumpy.

(e) Whether or not Ralph is happy, Quentin is being obnoxious.

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3. (10 marks)

*Suppose at least one of the following two statements is false:*

If Fred cannot lift the weight, then John can't lift it.

Although John can't lift the weight, Susan can lift it.

*Translate each of the two statements into SL, preserving as much structure as possible. Be sure to write down your translation scheme.*

Can Fred lift the weight?

4. (20 marks)

For each of the following:

Circle "tautology" if it is a WFF of SL that is a tautology.

Circle "contingent" if it is a contingent WFF of SL.

Circle "inconsistent" if it is an inconsistent WFF of SL.

Otherwise, don't circle anything.

QUESTIONS ON NEXT PAGE...

a.

$$((P \& Q) \& \sim(P \leftrightarrow Q))$$

inconsistent      contingent      tautology

b.

$$((P \vee \sim Q) \& \sim(\sim P \rightarrow \sim Q))$$

inconsistent      contingent      tautology

c.

$$(\sim P \rightarrow (P \rightarrow Q))$$

inconsistent      contingent      tautology

d.

$$(((P \rightarrow Q) \& (R \rightarrow S)) \rightarrow ((P \vee R) \rightarrow (Q \vee S)))$$

inconsistent      contingent      tautology

e.

$$(((P \& \sim Q) \vee R) \rightarrow ((R \& \sim Q) \rightarrow \sim P))$$

inconsistent      contingent      tautology

f.

$$(P \rightarrow \sim P)$$

inconsistent      contingent      tautology

g.

$$(P \rightarrow (Q \rightarrow P \ \& \ Q))$$

inconsistent          contingent          tautology

h.

$$(((P \ \& \ Q) \rightarrow R) \rightarrow ((P \rightarrow R) \ \& \ (Q \rightarrow R)))$$

inconsistent          contingent          tautology

i.

$$((P \leftrightarrow Q) \leftrightarrow \sim(P \leftrightarrow \sim Q))$$

inconsistent          contingent          tautology

j.

$$((Q \rightarrow R) \rightarrow ((P \rightarrow Q) \rightarrow (P \rightarrow R)))$$

inconsistent          contingent          tautology

5. (40 marks) For the following, if it is possible, show it using the SL natural deduction system from this course. If it is not possible, write “not derivable”

a.  $(P \rightarrow (P \rightarrow Q)), P \vdash Q$

b.  $(A \leftrightarrow B), (B \leftrightarrow C) \vdash (A \leftrightarrow C)$

c.  $((P \& Q) \rightarrow R) \vdash (P \rightarrow R)$

d.  $(P \ \& \ (P \leftrightarrow Q)) \vdash (P \ \& \ Q)$

e.  $P \vdash (Q \rightarrow (P \ \& \ Q))$